**S2. A Proof of .**

In this section, we prove that the statistics Q and Y are not correlated. To prove it, we proceed as follows. That is, with. For this result, note that due to the fact that, , and . Consequently, . In other words,  Hence,  and .